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Foundations · market mechanics from the ground up

The Greeks: delta, gamma, theta, vega, vanna and charm

Chapter 7 of 10

The price of an option depends on the underlying price, on time and on volatility. The Greeks measure how strongly. This chapter explains delta, gamma, theta, vega, vanna and charm, each with an intuitive meaning, the formula and a numerical example on the S&P 500, shows why gamma and theta rise towards expiry and derives the relation between gamma and theta.

Field	Value
Series	Futures, options and who makes the price
Scope	Chapter 7 of 10
Type	Explanation with worked examples, not a measurement
As of	11 November 2026
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Online	kruegeralgorithms.com/en/research/option-greeks-delta-gamma-theta-vega-vanna-charm

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In one sentence: The Greeks say by how much the price of an option changes when the underlying price, time or volatility changes, and two of them, vanna and charm, say how the delta shifts in the process, that is, the dealer's hedge.

What this chapter is: a reference chapter with formulas and worked examples. All values are computed with the Black-Scholes formula (S&P 500 at 7,000, 30 days, 16 % volatility, zero interest), not observed in the market. Every formula is checked against a direct recomputation of the price. **Not a trading recommendation.**

Chapter 6 derived the price of an option. But a dealer wants to know more than what his options are worth today. He wants to know what happens to their value if something changes tomorrow.

1. What the sensitivities are for

The price of a call depends on three quantities that change all the time: the index level S, the time to expiry T and the implied volatility σ. For each there is a number that says how sensitively the price reacts to it. They carry Greek letters and are therefore called the Greeks.

Greek	Question it answers
Delta	By how much does the price change when the index rises by one point?
Gamma	By how much does the delta change in the process?
Theta	How much does the option lose in one day if nothing else happens?
Vega	By how much does the price change when volatility rises by one point?
Vanna	By how much does the delta change when volatility rises by one point?
Charm	By how much does the delta change when one day passes?

The first four describe the price. The last two describe the delta. They matter because the delta determines the dealer's hedge (chapter 8).

The example for all sections is three calls with 30 days to expiry at an index level of 7,000:

Strike	Position	Price in points
6,800	in the money	250.65
7,000	at the money	128.09
7,200	out of the money	53.67

In all formulas d_1 and d_2 stand for the two quantities from chapter 6, $N(\cdot)$ for the distribution function of the normal distribution and $\varphi(\cdot)$ for its density, the bell curve.

2. Delta: how closely the option follows the index

Meaning. Delta is the price change of the option per index point. A delta of 0.51 means: if the index rises by one point, the call rises by 0.51 points. At the same time delta is the number of index units with which the option can be replicated (chapter 6).

Formula.

$$\Delta_{\text{call}} = N(d_1), \qquad \Delta_{\text{put}} = N(d_1) - 1$$

The delta of a call lies between 0 and 1, that of a put between −1 and 0. The difference is always exactly 1. That is put-call parity from chapter 5 in another form.

Numbers.

Strike	Call delta	Put delta
6,800 (in the money)	0.744	−0.256
7,000 (at the money)	0.509	−0.491
7,200 (out of the money)	0.277	−0.723

A call deep in the money behaves almost like the index itself (delta close to 1). A call far out of the money hardly reacts (delta close to 0). At the money the delta is around one half.

3. Gamma: how fast the delta changes

Meaning. The delta is not fixed. If the index rises, a call becomes more likely to end with value, and its delta rises. Gamma measures by how much: the change in delta per index point. Gamma is thus the curvature of the price curve, that is, what distinguishes an option from a future. A future has delta 1 and gamma 0.

Formula.

$$\Gamma = \frac{\varphi(d_1)}{S \cdot \sigma \cdot \sqrt{T}}$$

Call and put with the same strike have the same gamma.

Numbers. At the money $\Gamma = 0.00124$ per point. Per 100 points that is 0.124. So if the index rises from 7,000 to 7,100, the delta of the call rises from 0.509 to around 0.63. The exact recomputation gives 0.630.

Two properties determine where gamma is large:

- **It is largest at the money.** There every point decides whether the option is worth something in the end. At 6,800 and 7,200 it is lower, at 0.00100 and 0.00104.
- **It grows towards expiry.** The denominator contains $\sqrt{T}$. The shorter the time to expiry, the larger the gamma at the money.

Time to expiry	Gamma at the money	Ratio to 30 days
30 days	0.00124	1.0
5 days	0.00304	2.4
1 day	0.00681	5.5

With one day to expiry the gamma at the money is 5.5 times as large as with 30 days. That is $\sqrt{30}$. An option that expires today has an even much larger gamma in the afternoon: its delta jumps between 0 and 1 as soon as the index crosses the strike. That is why options with very short terms play a role of their own in chapter 9.

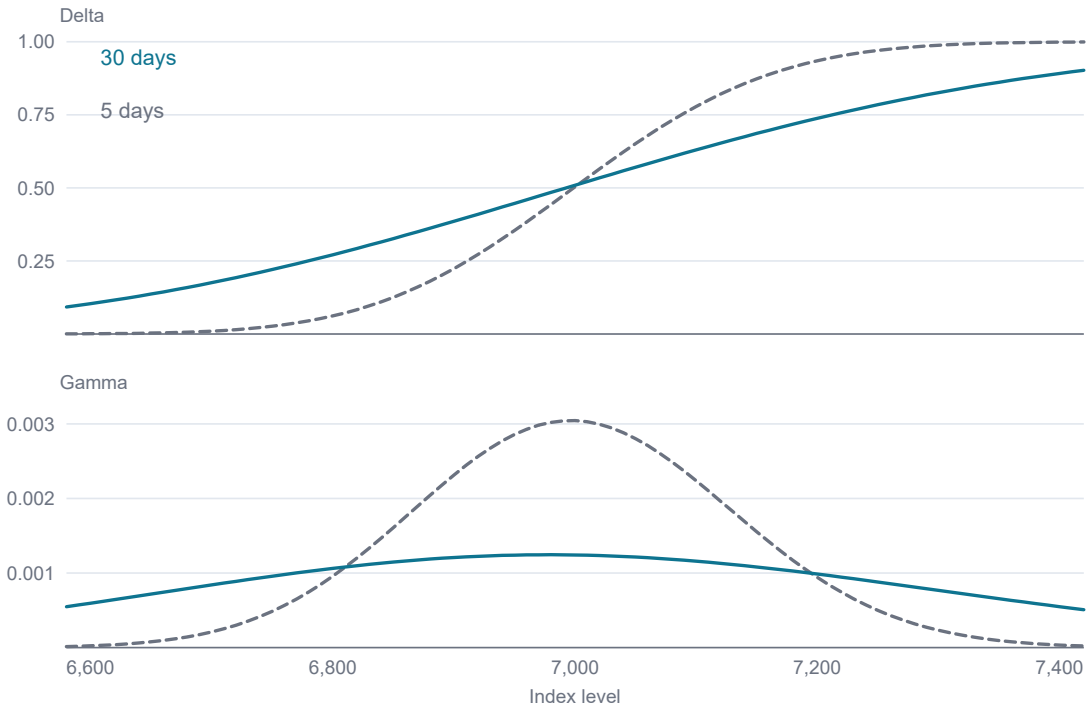


Fig. 1: Delta (top) and gamma (bottom) of a call with strike 7,000 by index level, at 30 days to expiry (solid) and at 5 days (dashed). Shortly before expiry the delta curve becomes steeper and gamma concentrates at the strike.

4. Theta: what a day costs

Meaning. Theta is the amount the option loses in one day if index and volatility do not change. It is the time value from chapter 5, melting away day by day.

Formula (without interest, per year; divide by 365 for one day).

$$\Theta = -\frac{S \cdot \varphi(d_1) \cdot \sigma}{2\sqrt{T}}$$

Numbers. At the money that is −779 points per year, that is −2.13 points per day. One contract loses 213 USD a day.

Time value does not melt evenly. It depends on $\sqrt{T}$, and a square root falls most steeply at the end:

Time to expiry	Price at the money	Theta per day
30 days	128.09	−2.13
5 days	52.30	−5.23
1 day	23.39	−11.69

In the first 25 days the option loses 76 points, in the last 5 days 52. The last day alone costs 23 points.

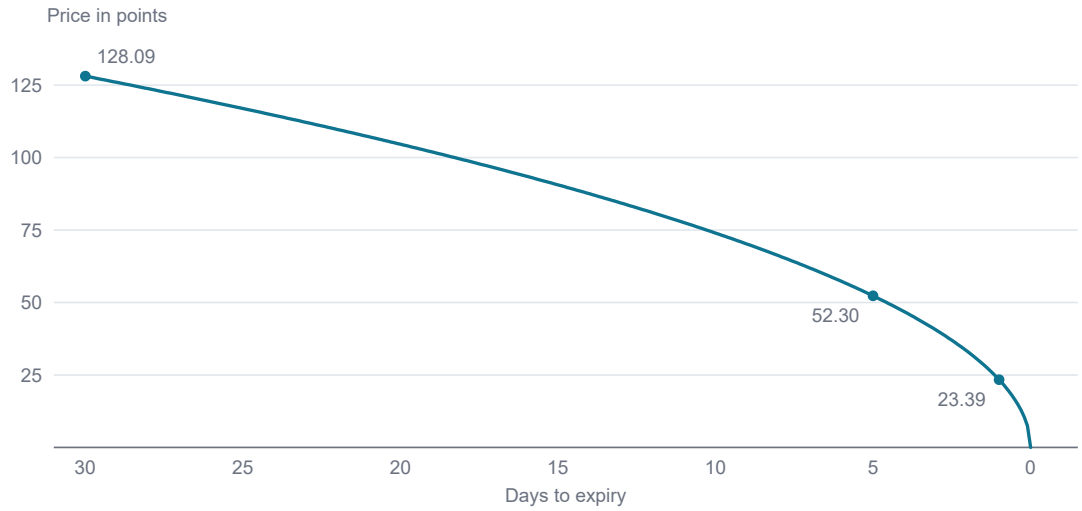


Fig. 2: Price of an at-the-money call with the index unchanged at 7,000 and 16 % volatility, from 30 days to expiry down to expiry. Time value melts faster the closer expiry comes.

5. Gamma and theta belong together

Gamma and theta are largest at the money and both grow towards expiry. That is no coincidence. If the formula for theta is divided by the one for gamma, almost everything drops out:

$$\frac{\Theta}{\Gamma} = -\frac{S \cdot \varphi(d_1) \cdot \sigma}{2\sqrt{T}} \cdot \frac{S \cdot \sigma \cdot \sqrt{T}}{\varphi(d_1)} = -\tfrac{1}{2}\,\sigma^2 S^2$$

So:

$$\Theta = -\tfrac{1}{2}\,\sigma^2 S^2\,\Gamma$$

The check with the numbers at the money:

$$\tfrac{1}{2} \times 0.16^2 \times 7{,}000^2 \times 0.00124 = 779 \text{ points per year}$$

Whoever holds gamma pays theta for it, in a fixed ratio. There is no such thing as a lot of gamma without a lot of theta. Here that is a rearrangement of two formulas. What lies behind it economically is shown in chapter 8: gamma brings money to whoever hedges his option on every price move, and theta is the price for that.

6. Vega: what a volatility point is worth

Meaning. Vega is the price change of the option when implied volatility rises by one point, for example from 16 % to 17 %. Whoever holds options earns from rising volatility. Whoever has sold them loses.

Formula (per 1.00 of volatility; divide by 100 for one volatility point).

$$\text{Vega} = S \cdot \varphi(d_1) \cdot \sqrt{T}$$

Numbers. At the money that is 8.00 points per volatility point, that is 800 USD per contract. This is the rule of thumb from chapter 6.

Vega behaves in the opposite way to gamma with respect to the term. In the formula $\sqrt{T}$ is in the numerator:

Time to expiry	Vega per volatility point	Gamma at the money
30 days	8.00	0.00124
5 days	3.27	0.00304
1 day	1.46	0.00681

Long-dated options react strongly to volatility and little to the individual price step. Short-dated ones react strongly to the price and hardly to volatility.

7. Vanna: how volatility shifts the delta

Meaning. Vanna is the change in delta when implied volatility rises by one point. The index does not move. The delta changes all the same, and with it the hedge a dealer needs.

Formula (per 1.00 of volatility; divide by 100 for one volatility point).

$$\text{Vanna} = -\frac{\varphi(d_1) \cdot d_2}{\sigma}$$

Numbers.

Strike	Delta at 16 % volatility	Vanna per volatility point	Delta at 17 %	Delta at 15 %
6,800 (in the money)	0.744	−0.0123	0.732	0.757
7,000 (at the money)	0.509	+0.0006	0.510	0.509
7,200 (out of the money)	0.277	+0.0133	0.290	0.263

The intuition behind it: more volatility means the index can travel further by expiry. For a call out of the money this makes it more likely to still get into the money. Its delta rises. For a call in the money it becomes less certain to stay there. Its delta falls. Rising volatility pulls all deltas towards the middle, falling volatility pushes them to the edges.

Why this matters: in quiet, rising markets implied volatility usually falls. The deltas of the options out of the money then shrink without the price having to move for it. Whoever has hedged such options has to adjust his hedge. Chapter 9 shows in which direction.

8. Charm: how time shifts the delta

Meaning. Charm is the change in delta when one day passes. Index and volatility stay the same.

Formula (without interest, per year; divide by 365 for one day).

$$\text{Charm} = \frac{\varphi(d_1) \cdot d_2}{2\,T}$$

Numbers.

Strike	Delta today	Charm per day	Delta tomorrow
6,800 (in the money)	0.744	+0.0033	0.747
7,000 (at the money)	0.509	−0.0002	0.509
7,200 (out of the money)	0.277	−0.0036	0.274

The intuition: on expiry day the delta is either 1 (in the money) or 0 (out of the money). It moves a little towards that with every day. A call out of the money loses delta, a call in the money gains it. Passing time acts on the delta like falling volatility: both push it to the edges.

Like gamma and theta, charm accelerates towards expiry, because T is in the denominator. In the last days before a large expiry the deltas therefore shift fastest, even if the price stands still.

9. All Greeks at a glance

The at-the-money call (strike 7,000, 30 days, 16 % volatility), per contract with 100 USD per point:

Greek	Value	Meaning in USD per contract
Price	128.09 points	12,809 USD
Delta	0.509	+50.90 USD per index point
Gamma	0.00124 per point	delta +0.124 per 100 points
Theta	−2.13 points per day	−213 USD per day
Vega	8.00 points per volatility point	+800 USD per volatility point
Vanna	+0.0006 per volatility point	almost zero at the money
Charm	−0.0002 per day	almost zero at the money

Vanna and charm are small at the money and large away from it. For gamma, theta and vega it is the other way round.

Common misconception

Common misconception: "Delta is the probability that the option ends in the money." As a rough estimate that will do, exactly right it is not. Delta is $N(d_1)$, the probability from the formula is $N(d_2)$. For the call with strike 7,200 these are 0.277 and 0.262. And $N(d_2)$ is not a prediction either: it is the risk-neutral computed quantity from chapter 6, which contains no view on direction. Delta is a hedging quantity, not a forecast.

What you should understand now

- Delta is the price change per index point and at the same time the number of index units in the replication. For a call it lies between 0 and 1, at the money around 0.5.
- Gamma is the change in delta per point. It is largest at the money and grows towards expiry with $1/\sqrt{T}$.
- Theta is the loss of time value per day. It accelerates towards expiry. Gamma and theta are firmly linked: $\Theta = -\frac{1}{2}\,\sigma^2 S^2\,\Gamma$.
- Vega is the price change per volatility point. It is large for long terms, gamma for short ones.
- Vanna and charm describe how volatility and time shift the delta. Falling volatility and passing time push the delta of out-of-the-money options towards zero.

Limits of this account

- **Small changes.** Each Greek holds for a small change in a single quantity. In reality price, time and volatility change at the same time, and all Greeks change with them.
- **Model.** The formulas assume Black-Scholes with a uniform volatility. With skew (chapter 6) volatility itself depends on the price, and the delta dealers actually use deviates from $N(d_1)$.
- **Without interest.** Theta and charm are given without interest and dividends. With interest small additional terms appear, the mechanics do not change.
- **Not covered:** rho (the effect of the interest rate) and further higher-order sensitivities.

Continue with chapter 8: what a dealer does with these numbers when he hedges his option book in the index.

An explanation of market mechanics with worked examples. Not investment advice and not a trading recommendation. Trading futures and options involves the risk of loss up to and beyond the capital employed.