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Foundations · market mechanics from the ground up

Hedging in the underlying: why option dealers with long gamma trade against the move and with short gamma trade with it

Chapter 8 of 10

An option dealer hedges his book with futures and has to adjust this hedge after every price move. With long gamma he sells into rising and buys into falling prices, with short gamma it is the other way round. Derived here: why the daily result of a hedged book is ½ · gamma · move² minus time decay, worked through on 100 calls on the S&P 500.

Field	Value
Series	Futures, options and who makes the price
Scope	Chapter 8 of 10
Type	Explanation with worked examples, not a measurement
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In one sentence: Whoever holds an option and offsets its price risk with futures has to trade again after every move, and whether he trades against the move or with it depends solely on the sign of his gamma.

What this chapter is: an explanation with a worked example, not a measurement. All numbers are computed (Black-Scholes, zero interest and dividends), not observed in the market. What of this mechanism shows up in real prices is in chapter 10. **Not a trading recommendation.**

This chapter needs three quantities from chapter 7. **Delta** says by how many points the option price changes when the index rises by one point. **Gamma** says by how much the delta changes in the process. **Theta** says how much the option loses in one day if nothing else happens.

1. What a dealer wants: the spread, not the direction

An option dealer (market maker) quotes buying and selling prices and earns the spread in between. A view on market direction is not part of his business. But every option a customer buys from him or sells to him puts exactly such a direction into his book. He gets rid of it by building the opposite position in the underlying. Underlying here means: what the option is written on, in practice the future on the index.

The example for this chapter:

Quantity	Value
S&P 500	7,000 points
Strike of the call	7,000 points
Time to expiry	30 days
Implied volatility	16 %
Price of the call	128.09 points
Price per contract (1 point = 100 USD)	12,809 USD
Delta	0.509
Gamma per point	0.00124
Theta per calendar day	−2.13 points

A customer sells the dealer 100 of these calls, for example a fund that writes calls on its share holdings. The dealer is now long 100 calls.

2. The delta hedge, step by step

Step 1: how strongly does the book depend on the index? One call gains 0.509 points when the index rises by one point. One point is 100 USD, and there are 100 contracts:

$$\begin{aligned}\text{Book delta} &= 0.509 \times 100 \times 100 \text{ USD} \\ &= 5{,}091 \text{ USD per index point}\end{aligned}$$

Step 2: what does he offset it with? With the E-mini future on the S&P 500. One future moves by 50 USD per index point.

$$\text{Futures} = \frac{5{,}091}{50} = 101.8 \approx 102 \text{ contracts, sold}$$

Step 3: the check. If the index rises by one point, the calls gain 5,091 USD and the 102 futures sold lose 5,100 USD. The sum is practically zero. The book is delta-neutral: small moves no longer change its value.

What is hedged is 5,091 USD per point times 7,000 points, that is around 35.6 million USD of index notional, for options that together cost 1.28 million USD.

3. The price moves: the hedge no longer fits

The hedge is only right for the price at which it was computed. If the index rises by 1 % to 7,070, the delta of the call rises. By how much is given by gamma:

$$\begin{aligned} \Delta_{\text{new}} &\approx \Delta + \Gamma \cdot dS \\ &= 0.509 + 0.00124 \times 70 = 0.596 \end{aligned}$$

The exact recomputation gives 0.595. The book delta is now 5,948 USD per point, and 119 futures sold are needed. The dealer has 102. He has to **sell 17 futures after the price has risen.**

If the index then falls back to 7,000, the delta falls back to 0.509. The dealer only needs 102 futures and **buys the 17 back after the price has fallen.** Sold at 7,070, bought back at 7,000:

$$17 \times 50 \text{ USD} \times 70 \text{ points} = 59{,}500 \text{ USD}$$

That is not trading skill. The dealer has only adjusted his hedge. The profit drops out because his delta runs along with the price: after a rise he is too long and has to sell, after a fall he is too short and has to buy. Selling high and buying back low is built into long gamma.

4. What the move brings in: ½ · gamma · move²

How large is this profit in general? If the index moves by dS points, the delta runs evenly from Δ to $\Delta + \Gamma \cdot dS$. So on average over the stretch it was

$$\bar{\Delta} = \Delta + \tfrac{1}{2}\Gamma \cdot dS$$

The option changes its value by average delta times distance:

$$dV \approx \bar{\Delta} \cdot dS = \Delta \cdot dS + \tfrac{1}{2}\Gamma \cdot dS^2$$

The hedge of futures sold brings $-\Delta \cdot dS$. The first part cancels. What remains:

$$\text{Gamma gain} = \tfrac{1}{2}\Gamma \cdot dS^2$$

Two things are in this formula. The move enters squared: whether the index rises or falls by 70 points is the same, and a move twice as large brings four times as much. And the sign comes from gamma alone.

For the example:

$$\begin{aligned} \tfrac{1}{2} \times 0.00124 \times 70^2 &= 3.04 \text{ points} \\ &= 304 \text{ USD per contract} \\ &= 30{,}432 \text{ USD for 100 contracts} \end{aligned}$$

The exact recomputation gives 30,163 USD. The difference is the error of the approximation, because gamma also changes slightly along the way. There and back it is twice around 30,400 USD. That fits the 59,500 USD from section 3, which are computed with whole futures.

5. What waiting costs: theta

If that were all, a hedged option would be a machine: every move brings money, whichever way. There is no such machine for free. The price for it is time decay. If the index stays at 7,000 for a day, each call loses 2.13 points, the book around 21,300 USD. Recomputed exactly it is 21,525 USD, because theta increases slightly in the course of the day.

How large this loss has to be can be derived. The implied volatility of 16 % is the fluctuation contained in the option price. It says how large the squared move is on average in a short period dt (in years):

$$\mathbb{E}\left[dS^2\right] = \sigma^2 S^2 \, dt$$

A fair price requires that a hedged book earns nothing on average if the market fluctuates exactly as much as priced in. So the average gamma gain and the time decay have to cancel:

$$\begin{aligned} \tfrac{1}{2}\,\Gamma\,\sigma^2 S^2\,dt + \Theta\,dt &= 0 \\ \Theta &= -\tfrac{1}{2}\,\sigma^2 S^2\,\Gamma \end{aligned}$$

That is the relation between gamma and theta. Whoever holds gamma pays theta for it, and the more so the higher implied volatility is. The check with the example numbers:

$$\begin{aligned} \tfrac{1}{2} \times 0.16^2 \times 7{,}000^2 \times 0.00124 &= 779 \text{ points per year} \\ &= 2.13 \text{ points per day} \end{aligned}$$

The two together give the daily result of a hedged book:

$$\text{Result} \approx \tfrac{1}{2}\,\Gamma\left(dS^2 - \sigma^2 S^2\,dt\right)$$

The bracket contains a comparison: the move that actually came against the move that was in the price. A hedged option book is therefore not a bet on direction. It is a bet on whether the market fluctuates more or less than priced in, that is, realised against implied volatility.

The break-even is where the bracket is zero:

$$\begin{aligned} |dS| = \sigma\,S\,\sqrt{dt} &= 0.16 \times 7{,}000 \times \sqrt{1/365} \\ &= 58.6 \text{ points} = 0.84\,\% \end{aligned}$$

If the index moves by more than 0.84 % in a day, the long-gamma book earns, below that it loses. The calculation here is in calendar days, because the time to expiry is given in calendar days. Counting in trading days (252 per year), the threshold is at 70.6 points or 1.01 %. That is the same calculation with a different calendar and exactly the quantity known as the expected move.

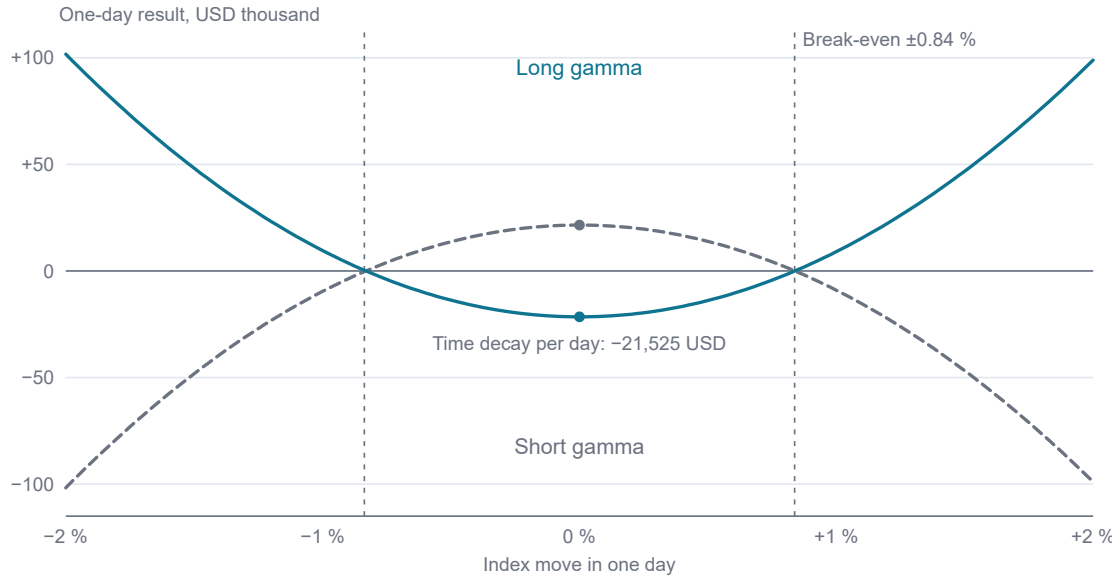


Fig. 1: Daily result of the hedged book of 100 calls by index move, recomputed exactly. Solid: the dealer holds the options (long gamma). Dashed: he has sold them (short gamma). Without a move the day costs or brings around 21,500 USD, the curves cross the zero line at ±0.84 %.

6. Five days worked through

The dealer hedges once a day at the closing price, with whole futures. The option is recomputed exactly every day.

Day	Index	Move	Delta	Futures sold	Trade	Options, USD	Futures, USD	Sum, USD
0	7,000		0.509	102	sells 102	0	0	0
1	7,070	+1.00 %	0.596	119	sells 17	+365,446	−357,000	+8,446
2	7,010	−0.85 %	0.522	104	buys 15	+8,102	0	+8,102
3	6,930	−1.14 %	0.417	83	buys 21	−389,697	+416,000	+26,303
4	6,990	+0.87 %	0.495	99	sells 16	−138,615	+167,000	+28,385
5	7,000	+0.14 %	0.508	102	sells 3	−111,584	+117,500	+5,916

The columns "Options" and "Futures" show the cumulative result since day 0.

Three things can be read off.

Every trade runs against the last move. After every rise he sells, after every fall he buys. The dealer does not decide that, his delta forces him to.

Large days earn, small days cost. Day 3 with −1.14 % brings around 18,200 USD, day 2 with −0.85 % lies almost exactly on the threshold and brings nothing, day 5 with +0.14 % costs around 22,500 USD.

In the end it is almost zero. After five days the index is back at 7,000, the options have lost 111,584 USD of time value, the futures have earned 117,500 USD. What remains is 5,916 USD. That is no coincidence: annualised, the five days fluctuated at 16.6 %, and 16 % was priced in. Realised and implied volatility were almost the same, so the book earns almost nothing.

The cross-check is a quiet week with the closing prices 7,015, 7,005, 6,990, 7,000 and 7,000. Annualised that is 3.1 % of fluctuation. The same book then loses 107,334 USD, almost exactly five times the daily time decay.

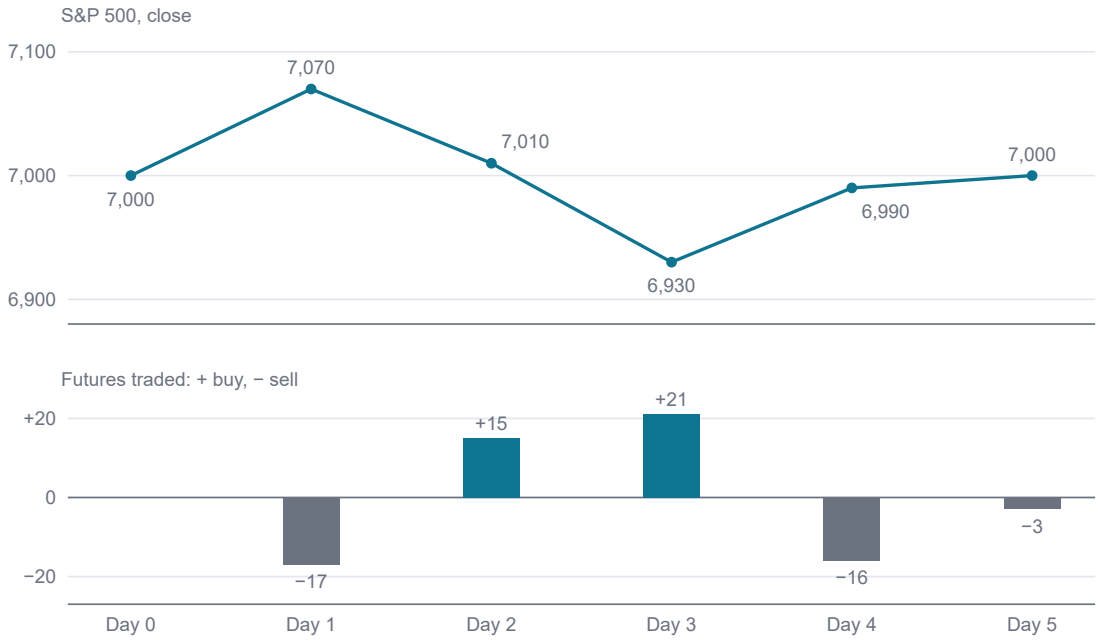


Fig. 2: On top the index path from the table, below it the futures the dealer trades at the close. Every bar points in the opposite direction to the day's move.

7. The other side: short gamma

Now a customer buys 100 puts with the same strike and the same expiry from the dealer, for example to protect his portfolio. The dealer has sold the puts and is short. Gamma and theta of these puts are as large as for the call, only the dealer carries them with the opposite sign.

A put here has a delta of −0.491. Puts sold have the opposite sign: the book gains 4,909 USD per index point to the upside. Against that the dealer sells 98 futures.

If the index falls by 1 % to 6,930, the put delta is −0.578. The book now depends on the index with 5,778 USD per point, and 116 futures sold are needed. The dealer has to **sell 18 futures after the price has fallen.** If the index rises again, he has to buy them back at a higher price.

Everything from sections 3 to 6 holds in mirror image:

	Long gamma	**Short gamma**
The dealer has	bought the options	sold the options
After a rise	he sells futures	he buys futures
After a fall	he buys futures	he sells futures
Effect on the price	dampening	amplifying
Move of 1 %	+30,400 USD	−30,400 USD
Day without a move	−21,300 USD	+21,300 USD
Earns if realised volatility	is above implied	is below implied

The amounts hold for the example book of 100 contracts and are rounded. Whether it is about calls or puts does not matter for this table. All that matters is whether the dealer holds the options or has sold them.

8. From the single book to the market

How much does a dealer have to trade after a move of 1 %? The move in points is $0.01 \cdot S$. The delta changes by $\Gamma \cdot 0.01 \cdot S$ per option. In USD of index notional, with N contracts and the multiplier m:

$$\text{Hedge flow per } 1\,\% = \Gamma \cdot S^2 \cdot 0.01 \cdot N \cdot m$$

For the example:

$$0.00124 \times 7{,}000^2 \times 0.01 \times 100 \times 100 = 6.09 \text{ million USD}$$
$$\approx 17 \text{ futures}$$

Those are the 17 futures from section 3: 6.09 million USD divided by the notional of one future of 350,000 USD (50 USD times 7,000 points). Summing this quantity over all strikes and expiries in which dealers hold positions gives the gamma exposure, GEX for short. Chapter 9 derives it and shows which assumptions it depends on. The most important one up front: the formula needs the sign of the dealer position, and exactly that is not publicly known.

The mechanics predict that a market in which dealers are short gamma overall has wider days. That can be measured. On a freely published gamma series for the S&P 500 the range of the next US session in the lower gamma tercile was 0.25 typical daily ranges larger than in the upper one (t 6.5), in the one-off holdout 0.20 (t 3.2). No direction could be derived from it (to the study).

Common misconception

> **Common misconception:** "Short gamma means the market falls." Gamma has no direction. A dealer with short gamma sells into falling prices and buys into rising ones. He amplifies the move that is running anyway, down as well as up. Short gamma is a statement about the width of a day, not about its sign.

A second misconception follows from the same confusion: that dealers "defend" a price level. The hedge is a reaction to a move that has already happened. Nobody decides in the process where the price should be.

What you should understand now

- A delta hedge makes an option book neutral only for small moves. After every larger move it has to be adjusted, and gamma determines the quantity needed.
- Whoever holds options (long gamma) sells after rises and buys after falls. Whoever has sold them (short gamma) does the opposite.
- A move of dS brings the long-gamma book $\frac{1}{2}\,\Gamma\,dS^2$, regardless of direction. Against that stands the daily time decay theta.
- Gamma and theta belong together: $\Theta = -\frac{1}{2}\,\sigma^2 S^2\,\Gamma$. The break-even of a day is the priced-in daily move $\sigma S\sqrt{dt}$.
- A hedged option book earns or loses on the difference between realised and implied volatility, not on market direction.

Limits of this account

- **Model.** The calculation uses Black-Scholes with constant volatility, without interest and dividends. In real markets implied volatility changes with the price, and with it the delta. This effect (vanna) and that of passing time on the delta (charm) are covered in chapter 7.
- **Hedging.** The dealer in the example hedges once a day and pays no costs. Real dealers hedge their whole book net, not every option individually, often only beyond a deviation and frequently with other options instead of futures. The flow in the underlying is therefore smaller and more irregular than the formula.
- **Approximation.** $\frac{1}{2}\,\Gamma\,dS^2$ holds for small moves. At 1 % it deviates from the exact value by just under 1 % in the example, for large jumps or shortly before expiry by considerably more.
- **Not shown:** who in the market actually holds which side. The example takes the dealer's position as known. In reality it is an estimate (chapter 9).

Continue with chapter 9: how this calculation is done for the whole market, and what open interest, GEX, the gamma flip and walls rest on.

An explanation of market mechanics with worked examples. Not investment advice and not a trading recommendation. Trading futures and options involves the risk of loss up to and beyond the capital employed.